# Riješiti sistem jednačina

$$\begin{aligned} x_1 + 2x_2 - 4x_3 + 16x_4 - 4x_5 &= -8 \\ -2x_1 - 3x_2 + 5x_3 - 20x_4 + 5x_5 &= 7 \\ 4x_1 + 9x_2 - 18x_3 + 72x_4 - 18x_5 &= -37 \end{aligned}$$

Rj.-upute:

Sistem ćemo riješiti Kroneker-Kapelijevom metodom

$$\bar{A} = [A \mid b] = \left[\begin{array}{ccccc|c} 1 & 2 & -4 & 16 & -4 & -8 \\ -2 & -3 & 5 & -20 & 5 & 7 \\ 4 & 9 & -18 & 72 & -18 & -37 \end{array}\right] \begin{array}{l} II_v + I_v \cdot 2 \\ III_v + I_v \cdot (-4) \end{array}$$

$$\ldots \sim \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & -4 & 1 & 4 \end{array}\right]$$

$$\Rightarrow \left.\begin{array}{l} \text{rang}(A) = 3 \\ \text{rang}(\bar{A}) = 3 \\ \text{broj nepoznatih} = 5 \end{array}\right\} \Rightarrow$$

sistem ima beskonačno mnogo rješenja i dvije promjenjive uzimamo proizvoljno npr. $x_4 = s$, $x_5 = t$

$$\begin{aligned} x_1 &= 2 \\ x_2 &= 3 \\ x_3 &= 4 + 4s - t \\ x_4 &= s \\ x_5 &= t \end{aligned}$$

$s, t \in \mathbb{R}$

(#) Riješiti sistem jednačina

$$
\begin{aligned}
x_1 + 2x_2 - 4x_3 - 4x_4 - 16x_5 &= -9\\
2x_1 + 5x_2 - 11x_3 - 11x_4 - 44x_5 &= -29\\
-4x_1 - 7x_2 + 14x_3 + 14x_4 + 56x_5 &= 30
\end{aligned}
$$

Rj.-upute:

Sistem ćemo riješiti Kronecker-Kapelijevom metodom

$$
\bar{A} = [A \,|\, b] = \left[\begin{array}{ccccc|c} 1 & 2 & -4 & -4 & -16 & -9\\ 2 & 5 & -11 & -11 & -44 & -29\\ -4 & -7 & 14 & 14 & 56 & 30 \end{array}\right] \begin{array}{c} II_v + I_v\cdot(-2) \\ III_v + I_v \cdot 4 \end{array}
$$

$$
\ldots \sim \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & 3\\ 0 & 1 & 0 & 0 & 0 & 4\\ 0 & 0 & 1 & 1 & 4 & 5 \end{array}\right].
$$

$$
\Rightarrow \left.\begin{array}{l} \text{rang}(A) = 3\\ \text{rang}(\bar{A}) = 3\\ \text{broj nepoznatih} = 5 \end{array}\right\} \Rightarrow
$$

sistem ima beskonačno mnogo rješenja i dvije promjenjive uzimamo proizvoljno npr. $x_4 = s$, $x_5 = t$

$$
\begin{aligned}
x_1 &= 3\\
x_2 &= 4\\
x_3 &= 5 - s - 4t\\
x_4 &= s\\
x_5 &= t
\end{aligned}
$$

$s, t \in \mathbb{R}$

(#) Riješiti sistem jednačina

$$\begin{aligned} x_1 + 2x_2 - 4x_3 - 8x_4 - 12x_5 &= -11 \\ -2x_1 - 3x_2 + 5x_3 + 10x_4 + 15x_5 &= 7 \\ -3x_1 - 5x_2 + 10x_3 + 20x_4 + 30x_5 &= 25 \end{aligned}$$

Rj.-upute:

Sistem ćemo riješiti Kroneker-Kapelijevom metodom

$$\bar{A} = [A \mid b] = \left[\begin{array}{ccccc|c} 1 & 2 & -4 & -8 & -12 & -11 \\ -2 & -3 & 5 & 10 & 15 & 7 \\ -3 & -5 & 10 & 20 & 30 & 25 \end{array}\right] \begin{array}{l} \\ II_v + I_v \cdot 2 \\ III_v + I_v \cdot 3 \end{array}$$

$$\ldots \sim \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & 0 & 6 \\ 0 & 0 & 1 & 2 & 3 & 7 \end{array}\right]$$

$$\Rightarrow \left.\begin{array}{l} \text{rang}(A) = 3 \\ \text{rang}(\bar{A}) = 3 \\ \text{broj nepoznatih} = 5 \end{array}\right\} \Rightarrow$$

sistem ima beskonačno mnogo rješenja i dvije promjenljive uzimamo proizvoljno npr. $x_4 = s$, $x_5 = t$

$x_1 = 5$

$x_2 = 6$

$x_3 = 7 - 2s - 3t$

$x_4 = s$

$x_5 = t$

$s, t \in \mathbb{R}$

(#) Riješiti sistem jednačina

$$\begin{aligned} x_1 + 2x_2 - 4x_3 + 8x_4 + 12x_5 &= -10 \\ 3x_1 + 7x_2 - 15x_3 + 30x_4 + 45x_5 &= -43 \\ -2x_1 - 3x_2 + 6x_3 - 12x_4 - 18x_5 &= 13 \end{aligned}$$

Rj.-upute:

Sistem ćemo riješiti Kroneker-Kapelijevom metodom

$$\bar{A} = [A \,|\, b] = \left[\begin{array}{ccccc|c} 1 & 2 & -4 & 8 & 12 & -10 \\ 3 & 7 & -15 & 30 & 45 & -43 \\ -2 & -3 & 6 & -12 & -18 & 13 \end{array}\right] \begin{array}{c} II_v + I_v \cdot (-3) \\ \sim \\ III_v + I_v \cdot 2 \end{array}$$

$$\ldots \sim \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 0 & 5 \\ 0 & 0 & 1 & -2 & -3 & 6 \end{array}\right]$$

$\Rightarrow$ $\left.\begin{array}{l} \text{rang}(A) = 3 \\ \text{rang}(\bar{A}) = 3 \\ \text{broj nepoznatih} = 5 \end{array}\right\} \Rightarrow$ sistem ima beskonačno mnogo rješenja i dvije promjenjive uzimamo proizvoljno npr. $x_4 = s$, $x_5 = t$, $s, t \in \mathbb{R}$

$$\begin{aligned} x_1 &= 4 \\ x_2 &= 5 \\ x_3 &= 6 + 2s + 3t \\ x_4 &= s \\ x_5 &= t \\ s, t &\in \mathbb{R} \end{aligned}$$

(#) Ispitati i nacrtati graf f-je $y = \dfrac{x-2}{x^2-8x+16}$

Rj.-upute:

DEFINICIONO PODRUČJE

$y = \dfrac{x-2}{(x-4)^2}$ $\quad x \neq 4$

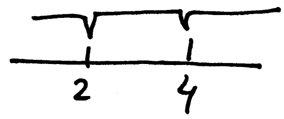
$D: x \in \mathbb{R} \setminus \{4\}$

$x \in (-\infty, 4) \cup (4, +\infty)$

ZNAK, NULE, PRESJEK SA y-OSOM

$(2; 0)$ je nula f-je

$(0; -\frac{1}{8})$ je presjek sa y-osom




← prekidi y + nule y

| x | $(-\infty, 2)$ | $(2,4)$ | $(4,+\infty)$ |
|---|---|---|---|
| y | − | + | + |

znak f-je

PARNOST (NEPARNOST), PERIODIČNOST

Definiciono područje nije simetrično pa f-ja nije ni parna ni neparna

PONAŠANJE NA KRAJEVIMA INTERVALA DEFINISANOSTI I ASIMPTOTE

vertikalna asimptota F-ja ima prekid za $x=4$

$$\lim_{x\to 4-0} f(x) = \lim_{x\to 4-0} \frac{x-2}{(x-4)^2} = \frac{4-0-2}{+0} = +\infty$$

$$\lim_{x\to 4+0} f(x) = \lim_{x\to 4+0} \frac{x-2}{(x-4)^2} = \frac{4+0-2}{+0} = +\infty$$

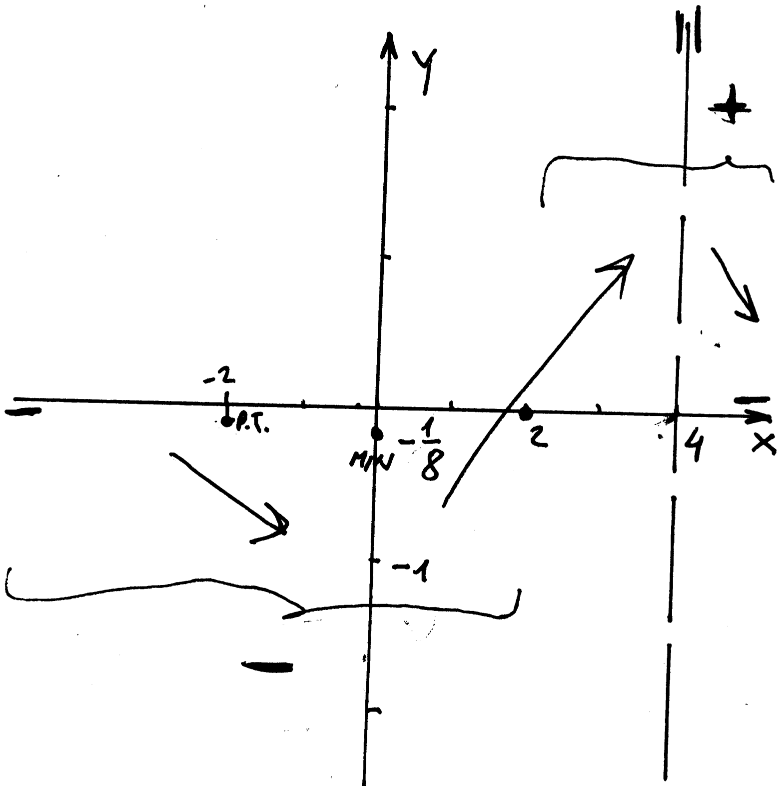
$\Rightarrow x=4$ je V.A.

horizontalna asimptota

$$\lim_{x\to \mp\infty} f(x) = \lim_{x\to \mp\infty} \frac{x-2 \;\; /:x}{x^2-8x+16 \;\; /:x} = 0 \Rightarrow y=0 \text{ je H.A.}$$

f-ja nema kosu asimptotu

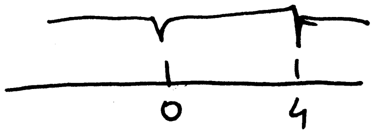
Poslije ovog koraka počinjemo skicirati graf f-je.




RAST I OPADANJE

$$y' = -\frac{x}{(x-4)^3}$$

prekidi y + nule y' →



| x | $(-\infty, 0)$ | $(0, 4)$ | $(4, +\infty)$ |
|---|---|---|---|
| $y'$ | $-$ | $+$ | $-$ |
| $y$ | ↘ | ↗ | ↘ |

MIN

intervali rasta i opadanja

$f(0) = -\frac{1}{8}$

EKSTREMI F-JE

Na osnovu tabele rasta i opadanja f-ja ima minimum u tački $(0; -\frac{1}{8})$.

PREVOJNE TAČKE I INTERVALI KONVEKSNOSTI I KONKAVNOSTI

$$y'' = \frac{2(x+2)}{(x-4)^4}$$

$-2$ $\quad$ $4$

✓ prekidi y + nule y''

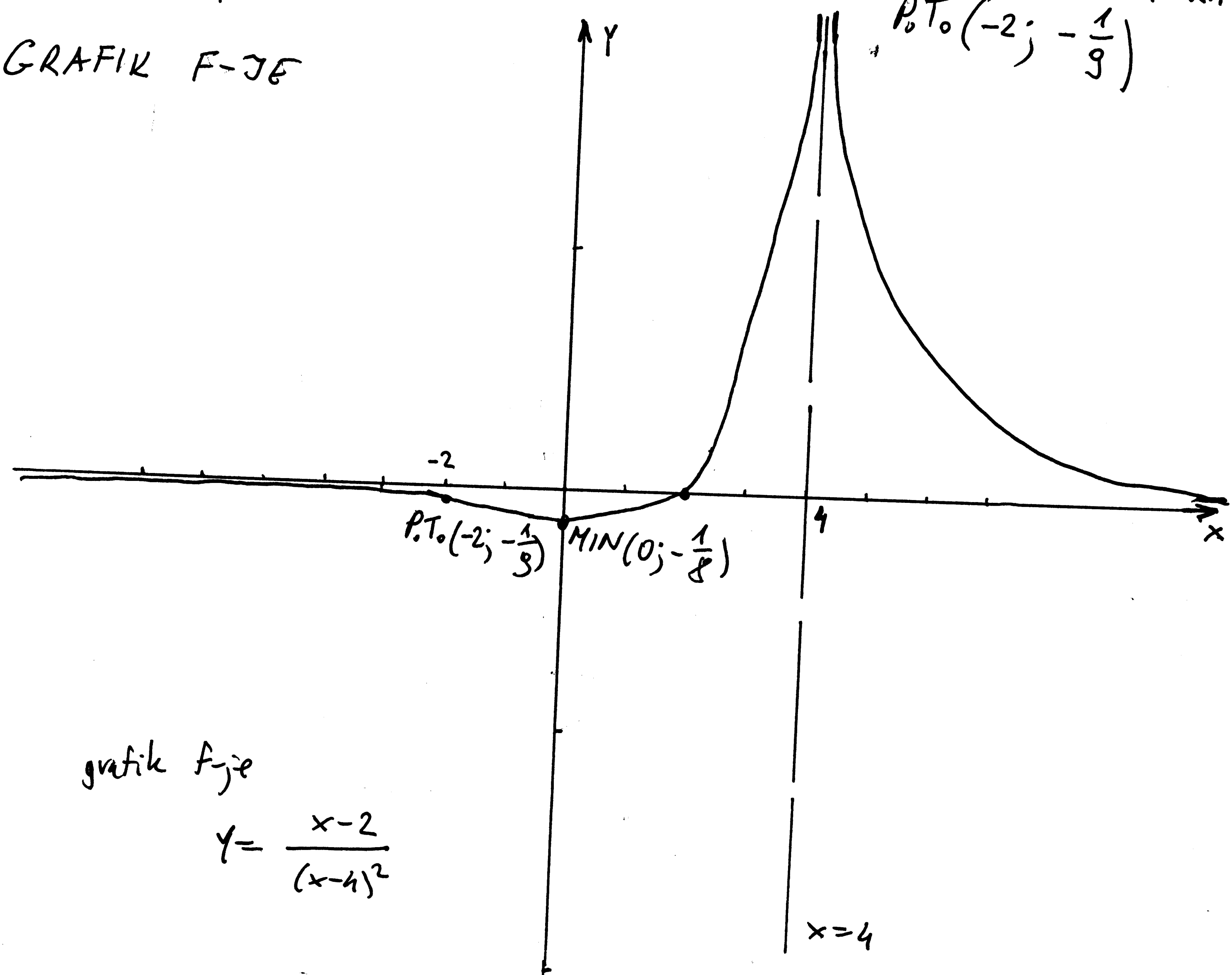
| x | $(-\infty, -2)$ | $(-2, 4)$ | $(4, +\infty)$ |
|---|---|---|---|
| $y''$ | $-$ | $+$ | $+$ |
| $y$ | $\cap$ | $\cup$ | $\cup$ |

$P_oT_o$

tabela konveksn. i konkavnosti

$P_oT_o(-2; -\frac{1}{9})$

GRAFIK F-JE




grafik f-je

$$y = \frac{x-2}{(x-4)^2}$$

(#) Ispitati i nacrtati graf f-je $y = \dfrac{x-5}{x^2-2x+1}$

Rj. -upute:

DEFINICIONO PODRUČJE

$y = \dfrac{x-5}{(x-1)^2}$ $\quad (x-1)^2 \neq 0$

$x-1 \neq 0$

$x \neq 1$

$\mathcal{D}: x \in \mathbb{R} \setminus \{1\}$

$x \in (-\infty, 1) \cup (1, +\infty)$

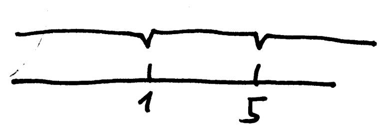
PARNOST, NEPARNOST, PERIODIČNOST

$\mathcal{D}$ nije simetrično $\Rightarrow$

f-ja nije ni parna ni neparna

f-ja nije periodična

prekid y + nule y




ZNAK, NULE, PRESJEK SA y-osom

$(5; 0)$ je nula f-je

$(0; -5)$ je presjek sa y-osom

| $x$ | $(-\infty, 1)$ | $(1, 5)$ | $(5, +\infty)$ |
|---|---|---|---|
| $y$ | $-$ | $-$ | $+$ |

znak f-je

PONAŠANJE NA KRAJEVIMA INTERVALA DEFINISANOSTI I ASIMPTOTE

vertikalna asimptota F-ja ima prekid za $x=1$

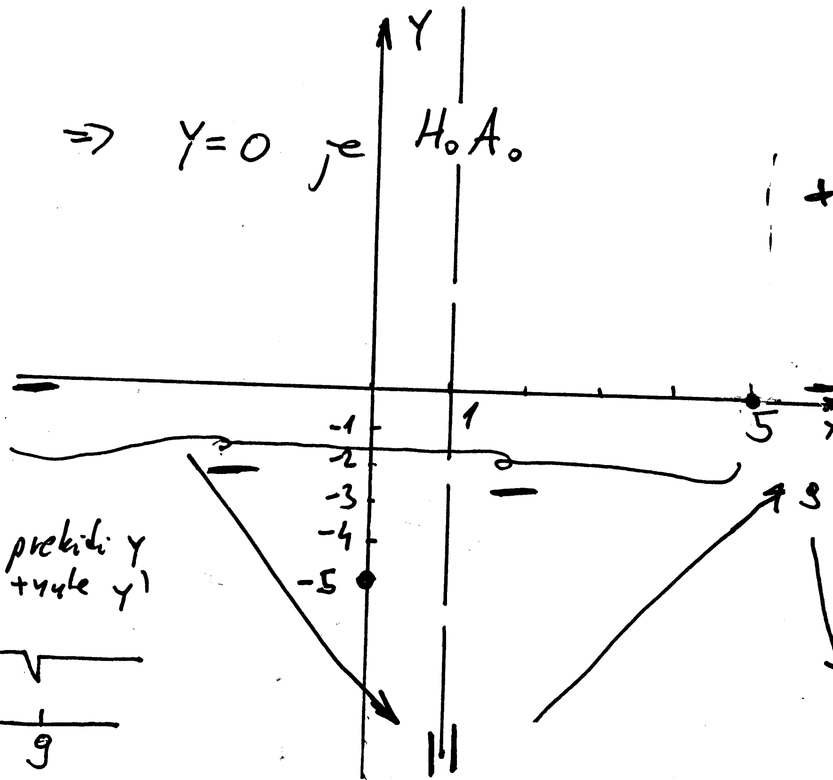
$$\left.\begin{aligned} &\lim_{x\to 1-0} f(x) = \lim_{x\to 1-0} \frac{x-5}{(x-1)^2} = \frac{1-0-5}{+0} = -\infty \\ &\lim_{x\to 1+0} f(x) = \lim_{x\to 1+0} \frac{x-5}{(x-1)^2} = \frac{1+0-5}{+0} = -\infty \end{aligned}\right\} \Rightarrow x=1 \text{ je V. A.}$$

horizontalna asimptota

$$\lim_{x\to \pm\infty} f(x) = \lim_{x\to \pm\infty} \frac{x-5 \quad /:x}{x^2-2x+1 \quad /:x} = 0 \Rightarrow y=0 \text{ je H. A.}$$

f-ja nema kosu asimptotu

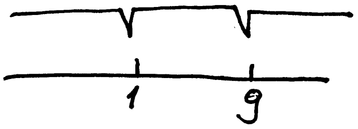
Poslije ovog koraka počinjemo skicirati graf f-je.




RAST I OPADANJE

$$y' = -\frac{x-9}{(x-1)^3}$$

prekidi y + nule y'



| x | $(-\infty, 1)$ | $(1, 9)$ | $(9, +\infty)$ |
|---|---|---|---|
| $y'$ | $-$ | $+$ | $-$ |
| $y$ | ↘ | ↗ | ↘ |

MAX

tabela rasta i opadanja

$$f(9) = \frac{9-5}{(9-1)^2} = \frac{4}{64}$$

$$f(9) = \frac{1}{16}$$

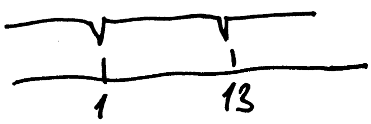
EKSTREMI F-JE

Na osnovu tabele rasta i opadanja f-ja ima ekstrem i to maksimum u tački $M(9; \frac{1}{16})$

PREVOJNE TAČKE I INTERVALI KONVEKSNOSTI I KONKAVNOSTI

$$y'' = 2\,\frac{x-13}{(x-1)^4}$$

prekidi y + nule y''



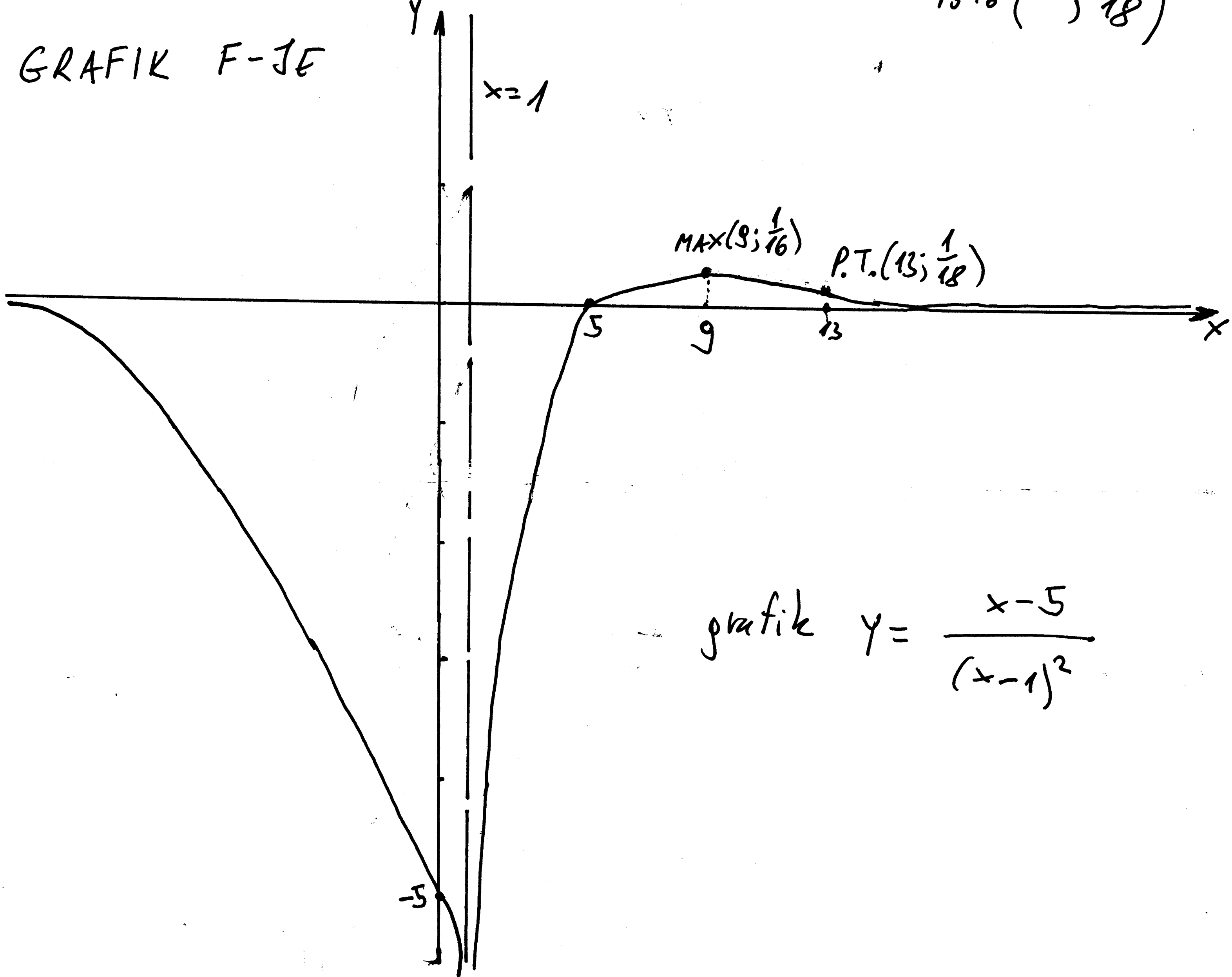

| x | $(-\infty, 1)$ | $(1, 13)$ | $(13, +\infty)$ |
|---|---|---|---|
| $y''$ | $-$ | $-$ | $+$ |
| $y$ | ∩ | ∩ | ∪ |

P.T.

tabela konveksnosti i konkavnosti

$$P.T.\left(13, \frac{1}{18}\right)$$

GRAFIK F-JE




grafik $y = \frac{x-5}{(x-1)^2}$

# (#) Ispitati i nacrtati graf f-je $y = \dfrac{x-3}{x^2-4x+4}$

Rj.-upute

## DEFINICIONO PODRUČJE

$$Y = \frac{x-3}{(x-2)^2} \qquad x \neq 2$$

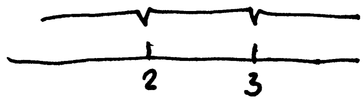
$$\mathcal{D}: x \in \mathbb{R} \setminus \{2\}$$

$$x \in (-\infty, 2) \cup (2, +\infty)$$

## ZNAK, NULE, PRESJEK SA Y-osom

$(3;0)$ je nula f-je

$(0; -\frac{3}{4})$ je presjek f-je sa Y-osom




prekidi y

+nule Y

| x | $(-\infty, 2)$ | $(2, 3)$ | $(3, +\infty)$ |
|---|---|---|---|
| Y | − | − | + |

znak f-je

## PARNOST (NEPARNOST), PERIODIČNOST

Definiciono područje nije simetrično pa f-ja nije ni parna ni neparna

F-ja nije periodična

## PONAŠANJE NA KRAJEVIMA INTERVALA DEFINISANOSTI I ASIMPTOTE

vertikalna asimptota F-ja ima prekid za $x = 2$

$$\lim_{x \to 2-0} f(x) = \lim_{x \to 2-0} \frac{x-3}{(x-2)^2} = \frac{2-0-3}{+0} = -\infty$$

$$\lim_{x \to 2+0} f(x) = \lim_{x \to 2+0} \frac{x-3}{(x-2)^2} = \frac{2+0-3}{+0} = -\infty$$

$\Rightarrow x = 2$ je V.A.

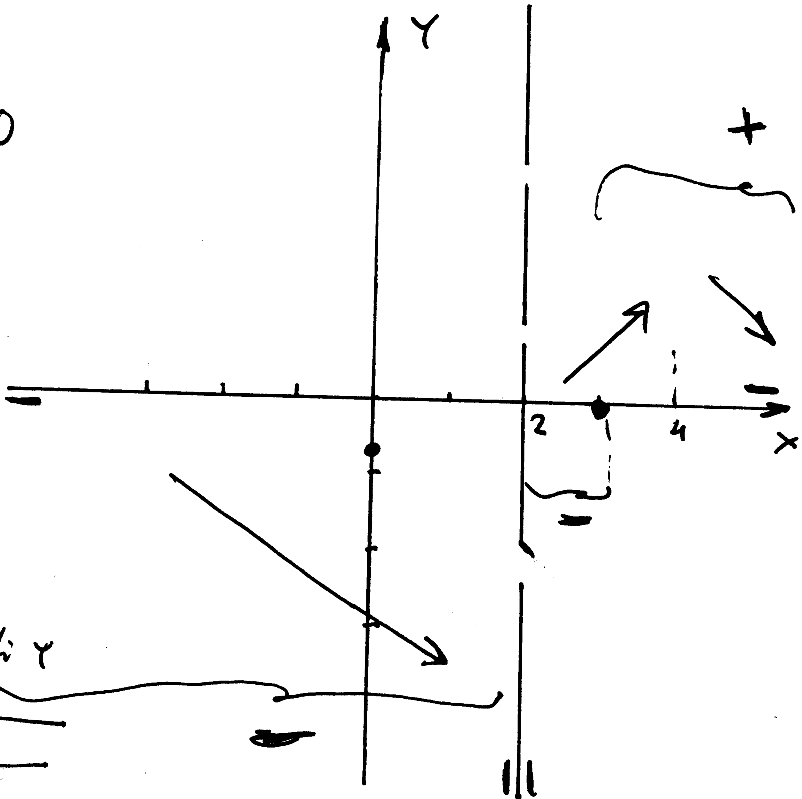
horizontalna asimptota

$$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{x-3}{x^2-4x+4} \begin{matrix} /:x \\ /:x \end{matrix} = 0$$

$\Rightarrow Y = 0$ je H.A.

f-ja nema kosu asimptotu.

Poslije ovog koraka počinjemo sa skiciranjem grafa f-je.

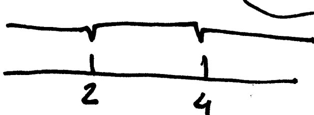



## RAST I OPADANJE

$$Y' = -\frac{x-4}{(x-2)^3}$$

nule y'

+ prekidi y



| $x$ | $(-\infty, 2)$ | $(2, 4)$ | $(4, +\infty)$ |
|---|---|---|---|
| $y'$ | $-$ | $+$ | $-$ |
| $y$ | $\searrow$ | $\nearrow$ | $\searrow$ |

MAX

tabela rasta i opadanja

$$f(4) = \frac{1}{4}$$

EKSTREMI F-JE

Na osnovu tabele rasta i opadanja f-ja ima maksimum u tački $M(4; \frac{1}{4})$.

PREVOJNE TAČKE I INTERVALI KONVEKSNOSTI I KONKAVNOSTI

$$y'' = \frac{2(x-5)}{(x-2)^4}$$

prekidi $y$ + nule $y''$

2 5

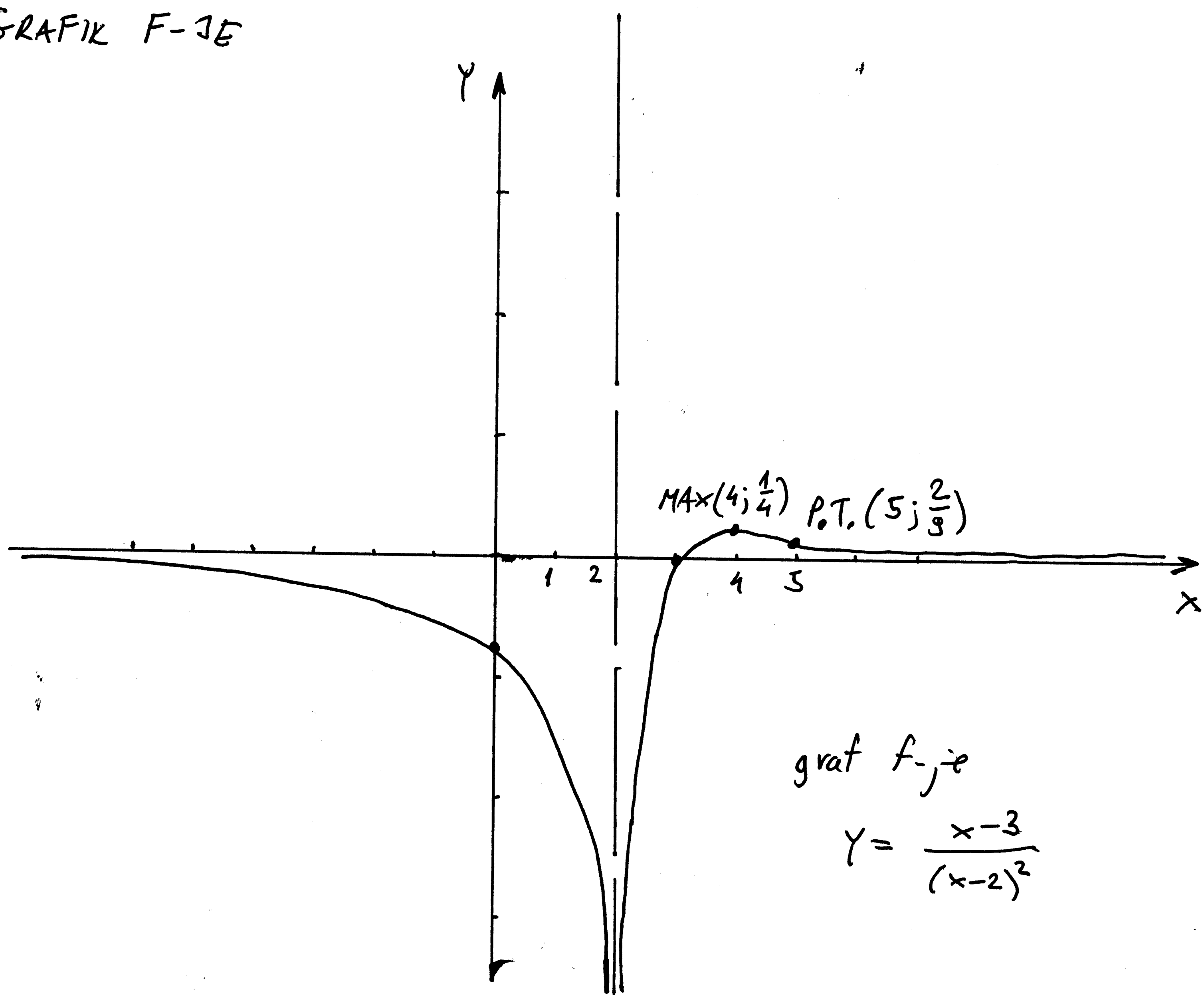
| $x$ | $(-\infty, 2)$ | $(2, 5)$ | $(5, +\infty)$ |
|---|---|---|---|
| $y''$ | $-$ | $-$ | $+$ |
| $y$ | $\frown$ | $\frown$ | $\smile$ |

P.T.

tabela konv. i konk.

$$P.T.\left(5, \frac{2}{9}\right)$$

GRAFIK F-JE




graf f-je

$$y = \frac{x-3}{(x-2)^2}$$

(#) Ispitati i nacrtati grafik f-je $Y = \dfrac{x-1}{x^2-10x+25}$

Rj.-upute

DEFINICIONO PODRUČJE

$Y = \dfrac{x-1}{(x-5)^2}$ $\quad (x-5)^2 \neq 0$

$x-5 \neq 0$

$x \neq 5$

$\mathcal{D}: x \in \mathbb{R} \setminus \{5\}$

$x \in (-\infty, 5) \cup (5, +\infty)$

ZNAK, NULE, PRESJEK SA Y-OSOM

$(1; 0)$ je nula f-je

$(0, -\frac{1}{25})$ je presjek sa Y-osom

↙ prekidi Y

+ nule Y

| x | $(-\infty, 1)$ | $(1, 5)$ | $(5, +\infty)$ |
|---|---|---|---|
| Y | − | + | + |

znak f-je

PARNOST (NEPARNOST), PERIODIČNOST

$\mathcal{D}$ nije simetrično $\Rightarrow$

$\Rightarrow$ f-ja nije ni parna ni neparna

PONAŠANJE NA KRAJEVIMA INTERVALA DEFINISANOSTI I ASIMPTOTE

vertikalna asimptota F-ja ima prekid za $x=5$

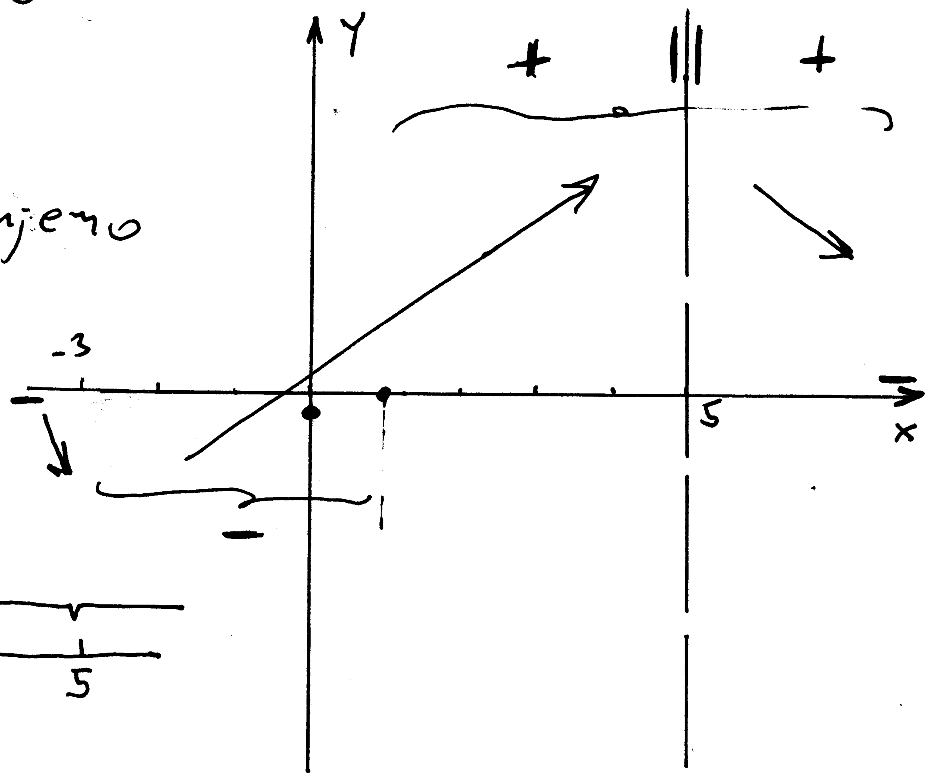
$$\left.\begin{aligned}\lim_{x\to 5-0} f(x) &= \lim_{x\to 5-0} \frac{x-1}{(x-5)^2} = \frac{5-0-1}{+0} = +\infty \\ \lim_{x\to 5+0} f(x) &= \lim_{x\to 5+0} \frac{x-1}{(x-5)^2} = \frac{5+0-1}{+0} = +\infty\end{aligned}\right\} \Rightarrow x=5 \text{ je V. A.}$$

horizontalna asimptota

$$\lim_{x\to \mp\infty} f(x) = \lim_{x\to \mp\infty} \frac{x-1 \quad /:x}{x^2-10x+25 \quad /:x} = 0 \Rightarrow Y=0 \text{ je H. A.}$$

f-ja nema kosu asimptotu

Poslije ovog koraka počinjemo skicirati graf f-je.




RAST I OPADANJE

$$Y' = -\frac{x+3}{(x-5)^3}$$

prekidi Y

+ nule Y'

-3 5

| x | $(-\infty,-3)$ | $(-3,5)$ | $(5,+\infty)$ |
|---|---|---|---|
| $y'$ | $-$ | $+$ | $-$ |
| $y$ | ↘ | ↗ | ↘ |
| | | MIN | |

tabela rasta i opadanja

$$f(-3) = -\frac{1}{16}$$

EKSTREMI F-JE

Na osnovu tabele rasta i opadanja f-ja ima minimum u tački $M(-3, -\frac{1}{16})$

PREVOJNE TAČKE I INTERVALI KONVEKSNOSTI I KONKAVNOSTI

$$y'' = \frac{2(x+7)}{(x-5)^4}$$

prekidi y + nule y''

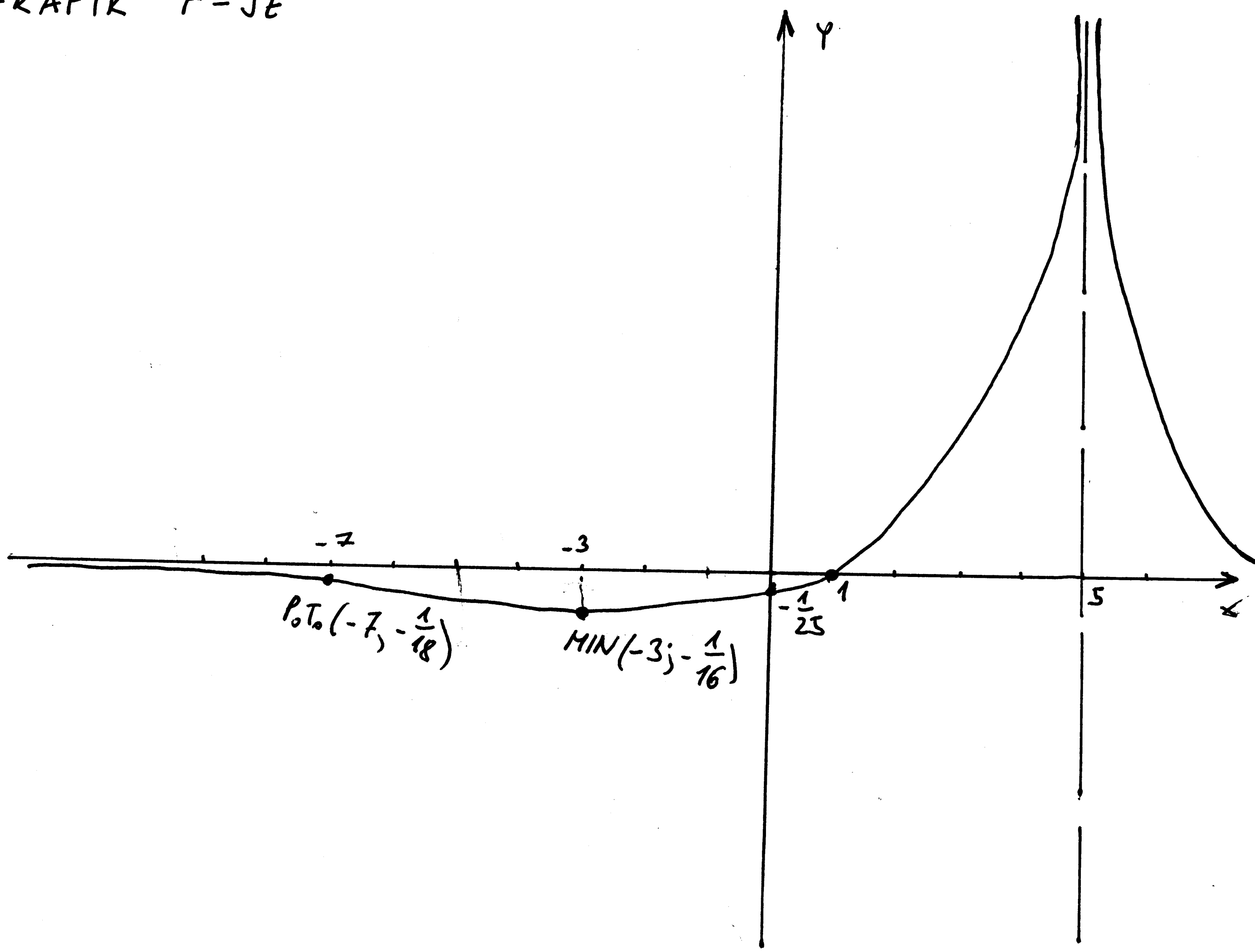
$-7$ $\quad$ $5$

| x | $(-\infty,-7)$ | $(-7,5)$ | $(5,+\infty)$ |
|---|---|---|---|
| $y''$ | $-$ | $+$ | $+$ |
| $y$ | $\cap$ | $\cup$ | $\cup$ |
| | | $P_oT_o$ | |

tabela konveksn. i konkavn.

$P_oT_o(-7, -\frac{1}{18})$

GRAFIK F-JE



# Izračunati integrale

a) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} x|\cos x|\,dx$ b) $\displaystyle\int_{0}^{2\pi} x|\sin x|\,dx$ c) $\displaystyle\int_{0}^{2\pi} e^{x}|\sin x|\,dx$ d) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} e^{x}|\cos x|\,dx$

Rj.

a)

$$|\cos x| = \begin{cases} -\cos x, & \cos x < 0 \\ \cos x, & \cos x \geq 0 \end{cases} = \begin{cases} \cos x, & x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ -\cos x, & x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \end{cases}$$

$$\int_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} x|\cos x|\,dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\cos x\,dx - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} x\cos x\,dx \overset{(*)}{=}$$

$$\boxed{\int x\cos x\,dx = \begin{vmatrix} u=x & dv=\cos x\,dx \\ du=dx & v=\sin x \end{vmatrix} = x\sin x - \int \sin x\,dx = x\sin x + \cos x}$$

$$\overset{(*)}{=} (x\sin x + \cos x)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\overset{\frac{\pi}{2}-\frac{\pi}{2}}{} - (x\sin x + \cos x)\Big|_{\pi/2}^{\frac{3\pi}{2}}\overset{-\frac{3\pi}{2}-\frac{\pi}{2}}{} = 0 - (-2\pi) = 2\pi$$

b)

$$|\sin x| = \begin{cases} -\sin x, & \sin x < 0 \\ \sin x, & \sin x \geq 0 \end{cases} = \begin{cases} \sin x, & x \in [0, \pi] \\ -\sin x, & x \in (\pi, 2\pi) \end{cases}$$

$$\boxed{\int x\sin x\,dx = \begin{vmatrix} u=x & dv=\sin x\,dx \\ du=dx & v=-\cos x \end{vmatrix} = -x\cos x + \int \cos x\,dx = \sin x - x\cos x}$$

$$\int_{0}^{2\pi} x|\sin x|\,dx = \int_{0}^{\pi} x\sin x\,dx - \int_{\pi}^{2\pi} x\sin x\,dx = (\sin x - x\cos x)\Big|_{0}^{\pi}\overset{\pi-0}{} - (\sin x - x\cos x)\Big|_{\pi}^{2\pi}\overset{-2\pi-\pi}{} =$$

$$= 4\pi$$

c) $$\int_0^{2\pi} e^x |\sin x|\, dx = \int_0^{\pi} e^x \sin x\, dx - \int_{\pi}^{2\pi} e^x \sin x\, dx \overset{(*)}{=}$$

$$I = \int e^x \sin x\, dx = \left| \begin{matrix} u = e^x & dv = \sin x\, dx \\ du = e^x dx & v = -\cos x \end{matrix} \right| = -e^x \cos x +$$

$$+ \int e^x \cos x\, dx = \left| \begin{matrix} u = e^x & dv = \cos x\, dx \\ du = e^x dx & v = \sin x \end{matrix} \right| = -e^x \cos x + e^x \sin x - \underbrace{\int e^x \sin x\, dx}_{I}$$

$$\Rightarrow \quad 2I = e^x(\sin x - \cos x) \quad \Rightarrow \quad I = \frac{1}{2} e^x (\sin x - \cos x)$$

$$\overset{(*)}{=} \frac{1}{2} e^x (\sin x - \cos x)\Big|_0^{\pi} - \frac{1}{2} e^x (\sin x - \cos x)\Big|_{\pi}^{2\pi} = \frac{1}{2}(e^{2\pi} + 2e^{\pi} + 1)$$

$$= \frac{1}{2}(e^{\pi} + 1)^2$$

d) $$\int_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} e^x |\cos x|\, dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^x \cos x\, dx - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} e^x \cos x\, dx =$$

$$= \frac{1}{2} e^x (\cos x + \sin x)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \frac{1}{2} e^x (\cos x + \sin x)\Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} =$$

$$= \frac{1}{2}\left(e^{\frac{\pi}{2}} + e^{-\frac{\pi}{2}}\right) - \frac{1}{2}\left(-e^{\frac{3\pi}{2}} - e^{\frac{\pi}{2}}\right) = \frac{1}{2}\left(e^{\frac{\pi}{2}} + e^{-\frac{\pi}{2}} + e^{\frac{3\pi}{2}} + e^{\frac{\pi}{2}}\right)$$

$$2e^{\frac{\pi}{2}} + e^{\frac{3\pi}{2}} + e^{-\frac{\pi}{2}}$$

$$= \frac{1}{2} e^{-\frac{\pi}{2}} \left( e^{\pi} + 2e^{\pi} + 1 \right) =$$

$$= \frac{1}{2} e^{-\frac{\pi}{2}} (e^{\pi} + 1)^2$$

(#) Riješiti diferencijalnu jednačinu

$$x^2(y+1)\,dx + y^2(x-1)\,dy = 0 .$$

Rj. Prisjetimo se: Za varijable jednačine $M(x,y)dx + N(x,y)dy = 0$ kažemo da su razdvojive ako se jednačina može napisati u obliku

$$f_1(x)\cdot g_2(y)\,dx + f_2(x)\cdot g_1(y)\,dy = 0$$

Ako ovu jednakost pomnožimo sa $\dfrac{1}{f_2(x)\,g_2(x)}$ dobićemo

$$\frac{f_1(x)}{f_2(x)}\,dx + \frac{g_1(y)}{g_2(y)}\,dy = 0$$

iz čega integraljenjem možemo dobiti primitivnu f-ju.

$x^2(y+1)\,dx + y^2(x-1)\,dy = 0$ ovo je dif. jedn. sa razdvojenim promjenjivim

$$/\cdot\frac{1}{(y+1)(x-1)}$$

$$\frac{x^2}{x-1}\,dx + \frac{y^2}{y+1}\,dy = 0$$

$$x^2 : (x-1) = x+1$$
$$\underline{-\ x^2 - x}$$
$$x$$
$$\underline{-\ x-1}$$
$$+1$$

$$\left(x+1+\frac{1}{x-1}\right)dx + \left(y-1+\frac{1}{y+1}\right)dy = 0 \quad /\int$$

$$y^2 : (y+1) = y-1$$
$$\underline{-\ y^2+y}$$
$$-y$$
$$\underline{-\ -y-1}$$
$$+1$$

$$\frac{1}{2}x^2 + x + \ln(x-1) + \frac{1}{2}y^2 - y + \ln(y+1) = C_1 \quad /\cdot 2$$

$$x^2 + y^2 + 2x - 2y + 2\ln(x-1)(y+1) = C_2$$

$$x^2 + 2\cdot x\cdot 1 + 1 - 1 + y^2 - 2\cdot y\cdot 1 + 1 - 1 + 2\ln(x-1)(y+1) = C_2$$

$$(x+1)^2 + (y-1)^2 + 2\ln(x-1)(y+1) = C$$ traženo rješenje opšte rješ. dif. jedn.

# Riješiti diferencijalnu jednačinu

$$4x\,dy - y\,dx = x^2\,dy$$

Rj.

$$4x\,dy - y\,dx = x^2\,dy \qquad / \cdot (-1)$$

$$y\,dx + (x^2 - 4x)\,dy = 0$$ ovo je diferencijalna jednačina sa razdvojenim promjenjivim

$$/ \cdot \frac{1}{(x^2-4x)\,y}$$

$$\frac{dx}{x^2-4x} + \frac{dy}{y} = 0$$

$$\frac{1}{x^2-4x} = \frac{1}{x(x-4)} = \frac{A}{x} + \frac{B}{x-4} \qquad / \cdot (x^2-4x)$$

$$1 = A(x-4) + Bx \quad \Rightarrow \quad \begin{aligned} A + B &= 0 \\ -4A &= 1 \end{aligned} \qquad A = -\frac{1}{4}, \quad B = \frac{1}{4}$$

$$\left( \frac{-\frac{1}{4}}{x} + \frac{\frac{1}{4}}{x-4} \right) dx + \frac{1}{y}\,dy = 0 \qquad / \int$$

$$-\frac{1}{4}\ln x + \frac{1}{4}\ln(x-4) + \ln y = C_1 \qquad / \cdot 4$$

$$-\ln x + \ln(x-4) + 4\ln y = \ln C$$

$$\ln\frac{x-4}{x} + \ln y^4 = \ln C$$

$$(x-4)\,y^4 = Cx$$ opšte rješ. dif. jedn. traženo rješenje

(#) Riješiti diferencijalnu jednačinu

$$\frac{dy}{dx} = \frac{4y}{x(y-3)}$$

Rj:

$$\frac{dy}{dx} = \frac{4y}{x(y-3)} \quad / \cdot dx \cdot \frac{y-3}{y}$$

$$\frac{y-3}{y} dy = \frac{4dx}{x}$$ ovo je diferencijalna jednač. sa razdvojenim promjenjivim

$$\left(1 - \frac{3}{y}\right) dy = \frac{4}{x} dx \quad / \int$$

$$y - 3\ln y = 4\ln x + \ln C_1$$

$$y = \ln x^4 + 3\ln y + \ln C_1$$

$$y = \ln\left(C_1 x^4 y^3\right)$$

$$C_1 x^4 y^3 = e^y$$

$$x^4 y^3 = Ce^y$$ opšte rješenje dif. jedn. traženo rješenje

# Odrediti partikularno rješenje diferencijalne jednačine $(1+x^3)dy - x^2 y\,dx = 0$ koje zadovoljava inicijalni uslov $x=1$, $y=2$.

Rj.

$(1+x^3)dy - x^2 y\,dx = 0$ ovo je diferencijalna jednačina sa razdvojenim promjenjivim

$$\Big/ \frac{1}{y(1+x^3)}$$

$$\frac{dy}{y} - \frac{x^2}{1+x^3}\,dx = 0 \quad \Big/ \int$$

$$\int \frac{dy}{y} - \frac{1}{3}\int \frac{d(1+x^3)}{1+x^3} = C_1$$

$$\ln y - \frac{1}{3}\ln(1+x^3) = \ln C_2 \quad / \cdot 3$$

$$3\ln y = \ln C(1+x^3)$$

$y^3 = C(1+x^3)$ opšte rješenje dif. jedn.

Za $x=1$, $y=2$: $\quad 2^3 = C(1+1) \quad \Rightarrow \quad C = 4$

$y^3 = 4(1+x^3)$ partikularno rješenje diferencijalne jednačine